

ABSTRACT

A topological index is a numerical parameter mathematically derived from the graph structure. In this paper, we introduce the modified multiplicative first and second neighborhood indices, multiplicative F -neighborhood index, general multiplicative neighborhood index, multiplicative inverse sum indeg neighborhood index, multiplicative harmonic neighborhood index and multiplicative symmetric division neighborhood index, first and second multiplicative Gourava neighborhood indices of a graph. We compute these newly defined multiplicative neighborhood indices for nanocones and dendrimers.

Keywords: multiplicative F -neighborhood index, multiplicative harmonic neighborhood index, multiplicative Gourava neighborhood indices, nanocone, dendrimer.

Mathematics Subject Classification: 05C05, 05C12, 05C90.

I. INTRODUCTION

A molecular graph is a graph such that its vertices represent to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry whose focus of interest is finding topological indices of a molecular graph which correlate well with chemical properties of the chemical molecules. Several topological indices have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR study, see [1, 2, 3].

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex u is the number of vertices adjacent to u and it is denoted by $d_G(u)$. Let $N(u) = \{v: uv \in E(G)\}$. Let $S_G(u) = \sum_{v \in N(u)} d_G(v)$ be the degree sum of neighbor vertices. For undefined term and notation, we refer the book [4].

Recently, some neighborhood indices were introduced and studied such as fifth M_1 and M_2 Zagreb indices [5], fifth hyper M_1 and M_2 Zagreb indices [6], general fifth M -Zagreb indices [6], F -neighborhood index, general first neighborhood index [7], fifth arithmetic-geometric index [8], fifth multiplicative Zagreb indices [9], fifth multiplicative hyper Zagreb indices [10], fifth multiplicative sum and product connectivity indices [10], general fifth multiplicative Zagreb indices [10], fourth multiplicative atom bond connectivity index [11], fifth multiplicative geometric-arithmetic index [9]. Also some neighborhood indices were studied in [12, 13, 14, 15, 16].

The fifth multiplicative M_1 and M_2 Zagreb indices (now we call multiplicative first and second neighborhood indices) were introduced by Kulli in 2017[9], defined as

$$NM_1II(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)], \quad NM_2II(G) = \prod_{uv \in E(G)} S_G(u)S_G(v).$$

The fifth multiplicative hyper Zagreb indices (now we call multiplicative first and second hyper neighborhood indices) were introduced by Kulli in 2018[10], defined as

$$HNM_1II(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]^2, \quad HNM_2II(G) = \prod_{uv \in E(G)} [S_G(u)S_G(v)]^2.$$

We introduce the modified multiplicative first and second neighborhood indices of a graph G and they are defined as

$${}^m NM_1 II(G) = \prod_{uv \in E(G)} \frac{1}{S_G(u) + S_G(v)}, \quad {}^m NM_2 II(G) = \prod_{uv \in E(G)} \frac{1}{S_G(u) S_G(v)}.$$

The fifth multiplicative sum connectivity index (now we call multiplicative neighborhood sum connectivity index) of a graph was introduced by Kulli in 2018[10], defined it as

$$SNMII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{S_G(u) + S_G(v)}}.$$

The fifth multiplicative sum connectivity index (now we call multiplicative neighborhood product connectivity index) of a graph was introduced by Kulli in 2018[10], defined it as

$$PNMII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{S_G(u) S_G(v)}}.$$

The general fifth multiplicative Zagreb indices (now we call multiplicative neighborhood sum and product connectivity indices) of a graph were introduced by Kulli in 2018[10], and they are defined as

$$NM_1^a II(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]^a,$$

$$NM_2^a II(G) = \prod_{uv \in E(G)} [S_G(u) S_G(v)]^a,$$

where a is a real numbers.

We now introduce the reciprocal multiplicative product connectivity index of a graph G , and it is defined as

$$RPNMII(G) = \prod_{uv \in E(G)} \sqrt{S_G(u) S_G(v)}.$$

We introduce some new multiplicative neighborhood indices as follows:

The multiplicative F - neighborhood index of a graph G is defined as

$$FNMII(G) = \prod_{uv \in E(G)} [S_G(u)^2 + S_G(v)^2].$$

The general multiplicative neighborhood index of a graph G is defined as

$$NM^a II(G) = \prod_{uv \in E(G)} [S_G(u)^a + S_G(v)^a].$$

We introduce the multiplicative inverse sum indeg neighborhood index of a graph G , defined as

$$INMII(G) = \prod_{uv \in E(G)} \frac{S_G(u) S_G(v)}{S_G(u) + S_G(v)}.$$

We propose the multiplicative harmonic neighborhood index of a graph G , defined it as

$$HNMII(G) = \prod_{uv \in E(G)} \frac{2}{S_G(u) + S_G(v)}.$$

We also introduce the multiplicative symmetric division neighborhood index of a graph G , defined as

$$SDNMII(G) = \prod_{uv \in E(G)} \left(\frac{S_G(u)}{S_G(v)} + \frac{S_G(v)}{S_G(u)} \right).$$

We propose first and second multiplicative Gourava neighborhood indices of a graph G , and they are defined as

$$NGO_1II(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v) + S_G(u)S_G(v)],$$

$$NGO_2II(G) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]S_G(u)S_G(v).$$

In this study, some new and old neighborhood indices of nanocones and 2 types of dendrimers are computed. For nanocones, dendrimers see 17.

2. RESULTS FOR NANOCONES $C_n[k]$

In this section, we consider nanocones $C_n[k]$. The molecular structure of $C_4[2]$ is shown in Figure 1.

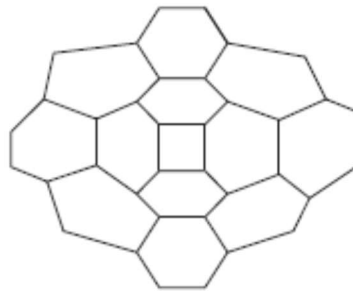


Figure 1. The molecular structure of $C_4[2]$

Let G be the molecular structure of $C_n[k]$. By calculation, G has $n(k+1)^2$ vertices and $\frac{n}{2}(k+1)(3k+2)$ edges. Also by calculation, we obtain that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degree of end vertices of each edge as given in Table 1.

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(5, 5)	n
(5, 7)	$2n$
(6, 7)	$2(k-1)n$
(7, 9)	nk
(9, 9)	$\frac{nk}{2}(3k-1)$

Table 1. Edge partition of $C_n[k]$ based on $S_G(u), S_G(v)$

We compute the multiplicative neighborhood general sum connectivity index of $C_n[k]$.

Theorem 1. The multiplicative neighborhood general sum connectivity index of nanocone $C_n[k]$ is

$$NM_1^a II(C_n[k]) = 10^{an} \times 12^{2an} \times 13^{2an(k-1)} \times 16^{ank} \times 18^{\frac{ank}{2}(3k-1)}. \tag{1}$$

Proof: Let G be the molecular graph of $C_n[k]$. By using the definition and Table 1, we deduce

$$NM_1^a II(C_n[k]) = \prod_{uv \in E(G)} [S_G(u) + S_G(v)]^a$$

$$= (5+5)^{an} \times (5+7)^{2an} \times (6+7)^{2an(k-1)} \times (7+9)^{ank} \times (9+9)^{\frac{ank}{2}(3k-1)}$$

$$= 10^{an} \times 12^{2an} \times 13^{2an(k-1)} \times 16^{ank} \times 18^{\frac{ank}{2}(3k-1)}.$$

The following results are obtained by using Theorem 1.

Corollary 1.1. The multiplicative first neighborhood index of $C_n[k]$ is

$$NM_1II(C_n[k]) = 10^n \times 12^{2n} \times 13^{2n(k-1)} \times 16^{nk} \times 18^{\frac{nk}{2}(3k-1)}.$$

Corollary 1.2. The multiplicative first hyper neighborhood index of $C_n[k]$ is

$$HNM_1II(C_n[k]) = 10^{2n} \times 12^{4n} \times 13^{4n(k-1)} \times 16^{2nk} \times 18^{nk(3k-1)}.$$

Corollary 1.3. The modified multiplicative first neighborhood index of $C_n[k]$ is

$${}^m NM_1II(C_n[k]) = \left(\frac{1}{10}\right)^n \times \left(\frac{1}{12}\right)^{2n} \times \left(\frac{1}{13}\right)^{2n(k-1)} \times \left(\frac{1}{16}\right)^{nk} \times \left(\frac{1}{18}\right)^{\frac{nk}{2}(3k-1)}.$$

Corollary 1.4. The multiplicative neighborhood sum connectivity index of $C_n[k]$ is

$$SNMII(C_n[k]) = \left(\frac{1}{\sqrt{10}}\right)^n \times \left(\frac{1}{12}\right)^n \times \left(\frac{1}{13}\right)^{n(k-1)} \times \left(\frac{1}{4}\right)^{nk} \times \left(\frac{1}{\sqrt{18}}\right)^{\frac{nk}{2}(3k-1)}.$$

Proof: Put $a = 1, 2, -1, -1/2$ in equation (1), we get the desired results.

We determine the multiplicative neighborhood general product connectivity index of $C_n[k]$.

Theorem 2. The multiplicative neighborhood general product connectivity index of nanocone $C_n[k]$ is

$$NM_2^a II(C_n[k]) = 25^{an} \times 35^{2an} \times 42^{2an(k-1)} \times 63^{ank} \times 81^{\frac{ank}{2}(3k-1)}. \quad (2)$$

Proof: By using the definition and Table 1, we derive

$$\begin{aligned} NM_2^a II(C_n[k]) &= \prod_{uv \in E(G)} [S_G(u)S_G(v)]^a \\ &= (5 \times 5)^{an} \times (5 \times 7)^{2an} \times (6 \times 7)^{2an(k-1)} \times (7 \times 9)^{ank} \times (9 \times 9)^{\frac{ank}{2}(3k-1)}. \\ &= 25^{an} \times 35^{2an} \times 42^{2an(k-1)} \times 63^{ank} \times 81^{\frac{ank}{2}(3k-1)}. \end{aligned}$$

We obtain the following results by using Theorem 2.

Corollary 2.1. The multiplicative second neighborhood index of $C_n[k]$ is

$$NM_2II(C_n[k]) = 25^n \times 35^{2n} \times 42^{2n(k-1)} \times 63^{nk} \times 81^{\frac{nk}{2}(3k-1)}.$$

Corollary 2.2. The multiplicative second hyper neighborhood index of $C_n[k]$ is

$$HNM_2II(C_n[k]) = 25^{2n} \times 35^{4n} \times 42^{4n(k-1)} \times 63^{2nk} \times 81^{nk(3k-1)}.$$

Corollary 2.3. The modified multiplicative second neighborhood index of $C_n[k]$ is

$${}^m NM_2II(C_n[k]) = \left(\frac{1}{25}\right)^n \times \left(\frac{1}{35}\right)^{2n} \times \left(\frac{1}{42}\right)^{2n(k-1)} \times \left(\frac{1}{63}\right)^{nk} \times \left(\frac{1}{81}\right)^{\frac{nk}{2}(3k-1)}.$$

Corollary 2.4. The multiplicative neighborhood product connectivity index of $C_n[k]$ is

$$PNMII(C_n[k]) = \left(\frac{1}{5}\right)^n \times \left(\frac{1}{35}\right)^n \times \left(\frac{1}{42}\right)^{n(k-1)} \times \left(\frac{1}{\sqrt{63}}\right)^{nk} \times \left(\frac{1}{9}\right)^{\frac{nk}{2}(3k-1)}.$$

Corollary 2.5. The reciprocal multiplicative neighborhood product connectivity index of $C_n[k]$ is

$$RPNMII(C_n[k]) = (5)^n \times (35)^n \times (42)^{n(k-1)} \times (\sqrt{63})^{nk} \times (9)^{\frac{nk}{2}(3k-1)}.$$

Proof: Put $a = 1, 2, -1, -1/2, 1/2$ in equation (2), we get the desired results.

Theorem 3. The general multiplicative neighborhood index of nanocone $C_n[k]$ is

$$NM^a II(C_n[k]) = (2 \times 5^a)^n \times (5^a + 7^a)^{2n} \times (6^a + 7^a)^{2n(k-1)} \times (7^a + 9^a)^{nk} \times (2 \times 9^a)^{\frac{nk}{2}(3k-1)}.$$

Proof: Let G be the molecular structure of $C_n[k]$. By using the definition and Table 1, we obtain

$$NM^a II(C_n[k]) = \prod_{uv \in E(G)} [S_G(u)^a + S_G(v)^a] \\ = (2 \times 5^a)^n \times (5^a + 7^a)^{2n} \times (6^a + 7^a)^{2n(k-1)} \times (7^a + 9^a)^{nk} \times (2 \times 9^a)^{\frac{nk}{2}(3k-1)}.$$

Corollary 3.1. The multiplicative F - neighborhood index of $C_n[k]$ is

$$FNMII(C_n[k]) = (50)^n \times (74)^{2n} \times (85)^{2n(k-1)} \times (130)^{nk} \times (162)^{\frac{nk}{2}(3k-1)}.$$

Theorem 4. The multiplicative inverse sum indeg neighborhood index of $C_n[k]$ is

$$INMII(C_n[k]) = \left(\frac{5}{2}\right)^n \times \left(\frac{35}{12}\right)^{2n} \times \left(\frac{42}{13}\right)^{2n(k-1)} \times \left(\frac{63}{16}\right)^{nk} \times \left(\frac{9}{2}\right)^{\frac{nk}{2}(3k-1)}.$$

Proof: Using the definition and Table 1, we have

$$INMII(C_n[k]) = \prod_{uv \in E(G)} \frac{S_G(u)S_G(v)}{S_G(u) + S_G(v)} \\ = \left(\frac{5 \times 5}{5 + 5}\right)^n \times \left(\frac{5 \times 7}{5 + 7}\right)^{2n} \times \left(\frac{6 \times 7}{6 + 7}\right)^{2n(k-1)} \times \left(\frac{7 \times 9}{7 + 9}\right)^{nk} \times \left(\frac{9 \times 9}{9 + 9}\right)^{\frac{nk}{2}(3k-1)} \\ = \left(\frac{5}{2}\right)^n \times \left(\frac{35}{12}\right)^{2n} \times \left(\frac{42}{13}\right)^{2n(k-1)} \times \left(\frac{63}{16}\right)^{nk} \times \left(\frac{9}{2}\right)^{\frac{nk}{2}(3k-1)}.$$

Theorem 5. The multiplicative harmonic neighborhood index of $C_n[k]$ is

$$HNMII(C_n[k]) = \left(\frac{1}{5}\right)^n \times \left(\frac{1}{6}\right)^{2n} \times \left(\frac{2}{13}\right)^{2n(k-1)} \times \left(\frac{1}{8}\right)^{nk} \times \left(\frac{1}{9}\right)^{\frac{nk}{2}(3k-1)}.$$

Proof: Using the definition and Table 1, we deduce

$$HNMII(C_n[k]) = \prod_{uv \in E(G)} \frac{2}{S_G(u) + S_G(v)} \\ = \left(\frac{2}{5 + 5}\right)^n \times \left(\frac{2}{5 + 7}\right)^{2n} \times \left(\frac{2}{6 + 7}\right)^{2n(k-1)} \times \left(\frac{2}{7 + 9}\right)^{nk} \times \left(\frac{2}{9 + 9}\right)^{\frac{nk}{2}(3k-1)} \\ = \left(\frac{1}{5}\right)^n \times \left(\frac{1}{6}\right)^{2n} \times \left(\frac{2}{13}\right)^{2n(k-1)} \times \left(\frac{1}{8}\right)^{nk} \times \left(\frac{1}{9}\right)^{\frac{nk}{2}(3k-1)}.$$

Theorem 6. The multiplicative symmetric division neighborhood index of $C_n[k]$ is

$$SDNMII(C_n[k]) = 2^{n + \frac{nk}{2}(3k-1)} \times \left(\frac{74}{35}\right)^{2n} \times \left(\frac{85}{42}\right)^{2n(k-1)} \times \left(\frac{130}{63}\right)^{nk}.$$

Proof: Using the definition and Table 1, we deduce

$$SDNMII(C_n[k]) = \prod_{uv \in E(G)} \left(\frac{S_G(u)}{S_G(v)} + \frac{S_G(v)}{S_G(u)}\right) \\ = \left(\frac{5}{5} + \frac{5}{5}\right)^n \times \left(\frac{5}{7} + \frac{7}{5}\right)^{2n} \times \left(\frac{6}{7} + \frac{7}{6}\right)^{2n(k-1)} \times \left(\frac{7}{9} + \frac{9}{7}\right)^{nk} \times \left(\frac{9}{9} + \frac{9}{9}\right)^{\frac{nk}{2}(3k-1)}$$

$$= 2^{n+\frac{nk}{2}(3k-1)} \times \left(\frac{74}{35}\right)^{2n} \times \left(\frac{85}{42}\right)^{2n(k-1)} \times \left(\frac{130}{63}\right)^{nk}.$$

Theorem 7. The first and second multiplicative Gourava neighborhood indices of nanocone $C_n[k]$ are given by

- (i) $NGO_1II(C_n[k]) = 35^n \times 47^{2n} \times 55^{2n(k-1)} \times 79^{nk} \times 99^{\frac{nk}{2}(3k-1)}.$
- (ii) $NGO_2II(C_n[k]) = 250^n \times 420^{2n} \times 546^{2n(k-1)} \times 1008^{nk} \times 1458^{\frac{nk}{2}(3k-1)}.$

Proof: By using the definition and Table 1, we deduce

$$\begin{aligned} \text{(i)} \quad NGO_1II(C_n[k]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v) + S_G(u)S_G(v)] \\ &= (5+5+5 \times 5)^{an} \times (5+7+5 \times 7)^{2an} \times (6+7+6 \times 7)^{2an(k-1)} \\ &\quad \times (7+9+7 \times 9)^{nk} \times (9+9+9 \times 9)^{\frac{ank}{2}(3k-1)}. \\ &= 35^n \times 47^{2n} \times 55^{2n(k-1)} \times 79^{nk} \times 99^{\frac{nk}{2}(3k-1)}. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad NGO_2II(C_n[k]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v)] S_G(u) S_G(v) \\ &= [(5+5)5 \times 5]^n + [(5+7)5 \times 7]^{2n} \times [(6+7)6 \times 7]^{2n(k-1)} \\ &\quad \times [(7+9)7 \times 9]^{nk} \times [(9+9)9 \times 9]^{\frac{ank}{2}(3k-1)}. \\ &= 250^n \times 420^{2n} \times 546^{2n(k-1)} \times 1008^{nk} \times 1458^{\frac{nk}{2}(3k-1)}. \end{aligned}$$

2. RESULTS FOR $NS_2[n]$ DENDRIMERS

In this section, we focus on the class of $NS_2[n]$ dendrimers with $n \geq 1$. The graph of $NS_2[2]$ is shown in Figure 2.

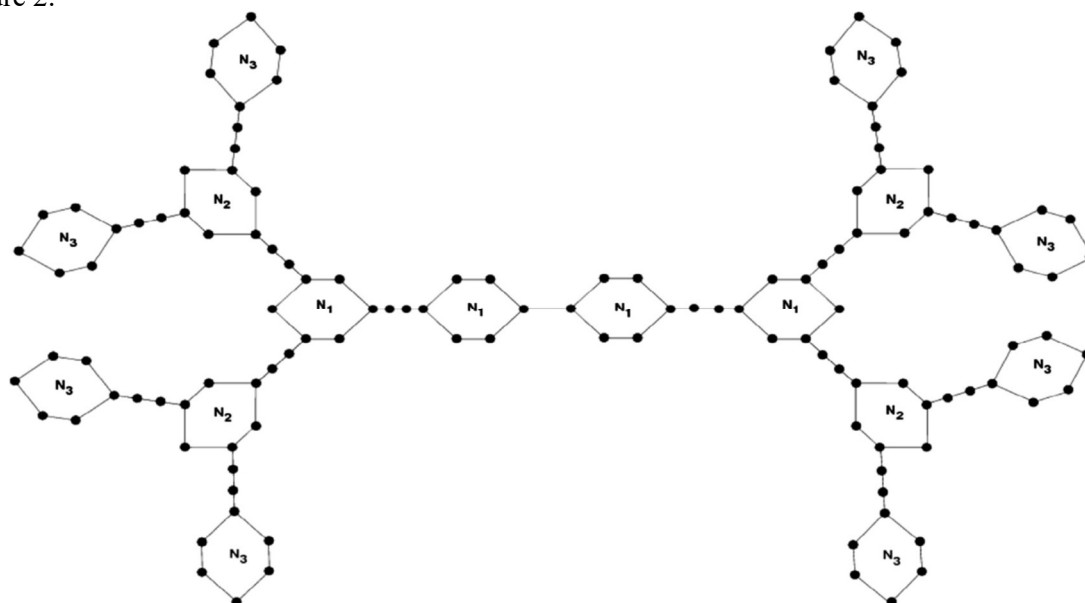


Figure 2. The graph of $NS_2[3]$

Let G be the graph of $NS_2[n]$. By calculation, G has $16 \times 2^n - 4$ vertices and $18 \times 2^n - 5$ edges. Also by calculation, we obtain that G has seven types of edges based on $S_G(u)$, $S_G(v)$ the degree of end vertices of each edge as given in Table 2.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4, 4)	(5, 4)	(5, 5)	(5, 6)	(7, 7)	(5, 7)	(6, 6)
Number of edges	2×2^n	2×2^n	$2 \times 2^n + 2$	6×2^n	1	4	$6 \times 2^n - 12$

Table 2. Edge partition of $NS_2[n]$ based on $S_G(u)$ and $S_G(v)$

We compute the multiplicative neighborhood general sum connectivity index of $NS_2[n]$.

Theorem 8. The multiplicative neighborhood general sum connectivity index of a dendrimer $NS_2[n]$ is

$$NM_1^a II(NS_2[n]) = 8^{a(2 \times 2^n)} \times 9^{a(2 \times 2^n)} \times 10^{a(2 \times 2^n + 2)} \times 11^{a(6 \times 2^n)} \times 14^a \times 12^{a(6 \times 2^n - 8)} \quad (3)$$

Proof: Let G be the molecular graph of $NS_2[n]$. Using definition and Table 2, we deduce

$$\begin{aligned} NM_1^a II(NS_2[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v)]^a \\ &= (4 + 4)^{a(2 \times 2^n)} \times (5 + 4)^{a(2 \times 2^n)} \times (5 + 5)^{a(2 \times 2^n + 2)} \times (5 + 6)^{a(6 \times 2^n)} \times (7 + 7)^a \times (5 + 7)^{4a} \times (6 + 6)^{a(6 \times 2^n - 8)} \\ &= 8^{a(2 \times 2^n)} \times 9^{a(2 \times 2^n)} \times 10^{a(2 \times 2^n + 2)} \times 11^{a(6 \times 2^n)} \times 14^a \times 12^{a(6 \times 2^n - 8)} \end{aligned}$$

The following results are obtained from Theorem 8.

Corollary 8.1. The multiplicative first neighborhood index of $NS_2[n]$ is

$$NM_1 II(NS_2[n]) = 8^{2 \times 2^n} \times 9^{2 \times 2^n} \times 10^{2 \times 2^n + 2} \times 11^{6 \times 2^n} \times 14 \times 12^{6 \times 2^n - 8}$$

Corollary 8.2. The multiplicative first hyper neighborhood index of $NS_2[n]$ is

$$HNM_1 II(NS_2[n]) = 8^{4 \times 2^n} \times 9^{4 \times 2^n} \times 10^{4 \times 2^n + 4} \times 11^{12 \times 2^n} \times 14^2 \times 12^{12 \times 2^n - 16}$$

Corollary 8.3. The modified multiplicative first neighborhood index of $NS_2[n]$ is

$${}^m NM_1 II(NS_2[n]) = \left(\frac{1}{8}\right)^{2 \times 2^n} \times \left(\frac{1}{9}\right)^{2 \times 2^n} \times \left(\frac{1}{10}\right)^{2 \times 2^n + 2} \times \left(\frac{1}{11}\right)^{6 \times 2^n} \times \frac{1}{14} \times \left(\frac{1}{12}\right)^{6 \times 2^n - 8}$$

Corollary 8.4. The multiplicative neighborhood sum connectivity index of $NS_2[n]$ is

$$SNM II(NS_2[n]) = \left(\frac{1}{\sqrt{8}}\right)^{2 \times 2^n} \times \left(\frac{1}{3}\right)^{2 \times 2^n} \times \left(\frac{1}{\sqrt{10}}\right)^{2 \times 2^n + 2} \times \left(\frac{1}{\sqrt{11}}\right)^{6 \times 2^n} \times \frac{1}{\sqrt{14}} \times \left(\frac{1}{\sqrt{12}}\right)^{6 \times 2^n - 8}$$

Proof: Put $a = 1, 2, -1, -1/2$ in equation (3), we obtain the desired results.

We compute the multiplicative neighborhood general product connectivity index of $NS_2[n]$.

Theorem 9. The multiplicative neighborhood general product connectivity index of dendrimer $NS_2[n]$ is

$$NM_2^a II(NS_2[n]) = 16^{a \times 2 \times 2^n} \times 20^{a \times 2 \times 2^n} \times 25^{a(2 \times 2^n + 2)} \times 30^{a \times 6 \times 2^n} \times 49^a \times 35^{4a} \times 36^{a(6 \times 2^n - 12)} \quad (4)$$

Proof: By using the definition and Table 2, we obtain

$$\begin{aligned} NM_2^a II(NS_2[n]) &= \prod_{uv \in E(G)} [S_G(u) S_G(v)]^a \\ &= (4 \times 4)^{a \times 2 \times 2^n} \times (5 \times 4)^{a \times 2 \times 2^n} \times (5 \times 5)^{a(2 \times 2^n + 2)} \times (5 \times 6)^{a \times 6 \times 2^n} \times (7 \times 7)^a \times (5 \times 7)^{4a} \times (6 \times 6)^{a(6 \times 2^n - 12)} \\ &= 16^{a \times 2 \times 2^n} \times 20^{a \times 2 \times 2^n} \times 25^{a(2 \times 2^n + 2)} \times 30^{a \times 6 \times 2^n} \times 49^a \times 35^{4a} \times 36^{a(6 \times 2^n - 12)} \end{aligned}$$

The following results are obtained by using Theorem 9.

Corollary 9.1. The multiplicative second neighborhood index of $NS_2[n]$ is

$$NM_2II(NS_2[n]) = 16^{2 \times 2^n} \times 20^{2 \times 2^n} \times 25^{2 \times 2^n + 2} \times 30^{6 \times 2^n} \times 49 \times 35^4 \times 36^{6 \times 2^n - 12}.$$

Corollary 9.2. The multiplicative second hyper neighborhood index of $NS_2[n]$ is

$$HNM_2II(NS_2[n]) = 16^{4 \times 2^n} \times 20^{4 \times 2^n} \times 25^{4 \times 2^n + 4} \times 30^{12 \times 2^n} \times 49^2 \times 35^8 \times 36^{12 \times 2^n - 24}.$$

Corollary 9.3. The modified multiplicative second neighborhood index of $NS_2[n]$ is

$${}^m NM_2II(NS_2[n]) = \left(\frac{1}{16}\right)^{2 \times 2^n} \times \left(\frac{1}{20}\right)^{2 \times 2^n} \times \left(\frac{1}{25}\right)^{2 \times 2^n + 2} \times \left(\frac{1}{30}\right)^{6 \times 2^n} \times \left(\frac{1}{49}\right) \times \left(\frac{1}{35}\right)^4 \times \left(\frac{1}{36}\right)^{6 \times 2^n - 12}.$$

Corollary 9.4. The multiplicative neighborhood product connectivity index of $NS_2[n]$ is

$$PNMII(NS_2[n]) = \left(\frac{1}{4}\right)^{2 \times 2^n} \times \left(\frac{1}{\sqrt{20}}\right)^{2 \times 2^n} \times \left(\frac{1}{5}\right)^{2 \times 2^n + 2} \times \left(\frac{1}{\sqrt{30}}\right)^{6 \times 2^n} \times \frac{1}{7} \times \left(\frac{1}{\sqrt{35}}\right)^4 \times \left(\frac{1}{6}\right)^{6 \times 2^n - 12}.$$

Corollary 9.5. The reciprocal multiplicative neighborhood product connectivity index of $NS_2[n]$ is

$$RPNMII(NS_2[n]) = 4^{2 \times 2^n} \times (\sqrt{20})^{2 \times 2^n} \times 5^{2 \times 2^n + 2} \times (\sqrt{30})^{6 \times 2^n} \times 7 \times (\sqrt{35})^4 \times (6)^{6 \times 2^n - 12}.$$

Proof: Put $a = 1, 2, -1, -1/2, 1/2$ in equation (4), we obtain the desired results.

Theorem 10. The general multiplicative neighborhood index of dendrimer $NS_2[n]$ is

$$NM^a II(NS_2[n]) = (2 \times 4^a)^{2 \times 2^n} \times (5^a + 4^a)^{2 \times 2^n} \times (2 \times 5^a)^{2 \times 2^n + 2} \times (5^a + 6^a)^{6 \times 2^n} \times (2 \times 7^a) \times (5^a + 7^a)^4 \times (2 \times 6^a)^{6 \times 2^n - 12}.$$

Proof: Using the definition and Table 2, we deduce

$$\begin{aligned} NM^a II(NS_2[n]) &= \prod_{uv \in E(G)} [S_G(u)^a + S_G(v)^a] \\ &= (2 \times 4^a)^{2 \times 2^n} \times (5^a + 4^a)^{2 \times 2^n} \times (2 \times 5^a)^{2 \times 2^n + 2} \times (5^a + 6^a)^{6 \times 2^n} \\ &\quad \times (2 \times 7^a) \times (5^a + 7^a)^4 \times (2 \times 6^a)^{6 \times 2^n - 12}. \end{aligned}$$

From Theorem 10, we establish the following result.

Corollary 10.1. The multiplicative F -neighborhood index of $NS_2[n]$ is

$$FNMII(NS_2[n]) = 32^{2 \times 2^n} \times 41^{2 \times 2^n} \times 50^{2 \times 2^n + 2} \times 61^{6 \times 2^n} \times 98 \times 74^4 \times 72^{6 \times 2^n - 12}.$$

Theorem 11. The multiplicative inverse sum indeg neighborhood index of $NS_2[n]$ is

$$INMII(NS_2[n]) = 2^{2 \times 2^n} \times \left(\frac{20}{9}\right)^{2 \times 2^n} \times \left(\frac{5}{2}\right)^{2 \times 2^n + 2} \times \left(\frac{30}{11}\right)^{6 \times 2^n} \times \frac{7}{2} \times \left(\frac{35}{12}\right)^4 \times 3^{6 \times 2^n - 12}.$$

Proof: By using the definition and Table 2, we deduce

$$\begin{aligned} INMII(NS_2[n]) &= \prod_{uv \in E(G)} \frac{S_G(u)S_G(v)}{S_G(u) + S_G(v)} \\ &= \left(\frac{4 \times 4}{4 + 4}\right)^{2 \times 2^n} \times \left(\frac{5 \times 4}{5 + 4}\right)^{2 \times 2^n} \times \left(\frac{5 \times 5}{5 + 5}\right)^{2 \times 2^n + 2} \times \left(\frac{5 \times 6}{5 + 6}\right)^{6 \times 2^n} \times \left(\frac{7 \times 7}{7 + 7}\right)^1 \times \left(\frac{5 \times 7}{5 + 7}\right)^4 \times \left(\frac{6 \times 6}{6 + 6}\right)^{6 \times 2^n - 12} \\ &= 2^{2 \times 2^n} \times \left(\frac{20}{9}\right)^{2 \times 2^n} \times \left(\frac{5}{2}\right)^{2 \times 2^n + 2} \times \left(\frac{30}{11}\right)^{6 \times 2^n} \times \frac{7}{2} \times \left(\frac{35}{12}\right)^4 \times 3^{6 \times 2^n - 12}. \end{aligned}$$

Theorem 12. The multiplicative harmonic neighborhood index of $NS_2[n]$ is

$$HNMI(NS_2[n]) = \left(\frac{1}{4}\right)^{2 \times 2^n} \times \left(\frac{2}{9}\right)^{2 \times 2^n} \times \left(\frac{1}{5}\right)^{2 \times 2^n + 2} \times \left(\frac{2}{11}\right)^{6 \times 2^n} \times \frac{1}{7} \times \left(\frac{1}{6}\right)^{6 \times 2^n - 8}.$$

Proof: By using the definition and Table 2, we derive

$$\begin{aligned} HNMI(NS_2[n]) &= \prod_{uv \in E(G)} \frac{2}{S_G(u) + S_G(v)} \\ &= \left(\frac{2}{4+4}\right)^{2 \times 2^n} \times \left(\frac{2}{5+4}\right)^{2 \times 2^n} \times \left(\frac{2}{5+5}\right)^{2 \times 2^n + 2} \times \left(\frac{2}{5+6}\right)^{6 \times 2^n} \times \left(\frac{2}{7+7}\right)^1 \times \left(\frac{2}{5+7}\right)^4 \times \left(\frac{2}{6+6}\right)^{6 \times 2^n - 12} \\ &= \left(\frac{1}{4}\right)^{2 \times 2^n} \times \left(\frac{2}{9}\right)^{2 \times 2^n} \times \left(\frac{1}{5}\right)^{2 \times 2^n + 2} \times \left(\frac{2}{11}\right)^{6 \times 2^n} \times \frac{1}{7} \times \left(\frac{1}{6}\right)^{6 \times 2^n - 8}. \end{aligned}$$

Theorem 13. The multiplicative symmetric division neighborhood index of $NS_2[n]$ is

$$SDNMI(NS_2[n]) = 2^{10 \times 2^n - 9} \times \left(\frac{41}{20}\right)^{2 \times 2^n} \times \left(\frac{61}{30}\right)^{6 \times 2^n} \times \left(\frac{74}{35}\right)^4.$$

Proof: Using the definition and Table 2, we obtain

$$\begin{aligned} SDNMI(NS_2[n]) &= \prod_{uv \in E(G)} \left(\frac{S_G(u)}{S_G(v)} + \frac{S_G(v)}{S_G(u)}\right) \\ &= \left(\frac{4}{4} + \frac{4}{4}\right)^{2 \times 2^n} \times \left(\frac{5}{4} + \frac{4}{5}\right)^{2 \times 2^n} \times \left(\frac{5}{5} + \frac{5}{5}\right)^{2 \times 2^n + 2} \times \left(\frac{5}{6} + \frac{6}{5}\right)^{6 \times 2^n} \times \left(\frac{7}{7} + \frac{7}{7}\right)^1 \times \left(\frac{5}{7} + \frac{7}{5}\right)^4 \times \left(\frac{6}{6} + \frac{6}{6}\right)^{6 \times 2^n - 12} \\ &= 2^{10 \times 2^n - 9} \times \left(\frac{41}{20}\right)^{2 \times 2^n} \times \left(\frac{61}{30}\right)^{6 \times 2^n} \times \left(\frac{74}{35}\right)^4. \end{aligned}$$

Theorem 14. The first and second multiplicative Gourava neighborhood indices of nanocone $NS_2[n]$ are

- (i) $NGO_1II(NS_2[n]) = 24^{2 \times 2^n} \times 29^{2 \times 2^n} \times 35^{2 \times 2^n + 2} \times 41^{6 \times 2^n} \times 63 \times 47^4 \times 48^{6 \times 2^n - 12}.$
- (ii) $NGO_2II(NS_2[n]) = 128^{2 \times 2^n} \times 180^{2 \times 2^n} \times 250^{2 \times 2^n + 2} \times 330^{6 \times 2^n} \times 686 \times 420^4 \times 432^{6 \times 2^n - 12}.$

Proof: By using the definitions and Table 2, we obtain

$$\begin{aligned} \text{(i)} \quad NGO_1II(NS_2[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v) + S_G(u)S_G(v)] \\ &= (4+4+4 \times 4)^{2 \times 2^n} \times (5+4+5 \times 4)^{2 \times 2^n} \times (5+5+5 \times 5)^{2 \times 2^n + 2} \times (5+6+5 \times 6)^{6 \times 2^n} \\ &\quad \times (7+7+7 \times 7)^1 \times (5+7+5 \times 7)^4 \times (6+6+6 \times 6)^{6 \times 2^n - 12}. \\ &= 24^{2 \times 2^n} \times 29^{2 \times 2^n} \times 35^{2 \times 2^n + 2} \times 41^{6 \times 2^n} \times 63 \times 47^4 \times 48^{6 \times 2^n - 12}. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad NGO_2II(NS_2[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v)] S_G(u) S_G(v) \\ &= [(4+4)4 \times 4]^{2 \times 2^n} \times [(5+4)5 \times 4]^{2 \times 2^n} \times [(5+5)5 \times 5]^{2 \times 2^n + 2} \times [(5+6)5 \times 6]^{6 \times 2^n} \\ &\quad \times [(7+7)7 \times 7]^1 \times [(5+7)5 \times 7]^4 \times [(6+6)6 \times 6]^{6 \times 2^n - 12}. \\ &= 128^{2 \times 2^n} \times 180^{2 \times 2^n} \times 250^{2 \times 2^n + 2} \times 330^{6 \times 2^n} \times 686 \times 420^4 \times 432^{6 \times 2^n - 12}. \end{aligned}$$

4. RESULTS FOR $NS_3[n]$ DENDRIMERS

In this section, we focus on another type of dendrimers $NS_3[n]$ with $n \geq 1$. The molecular structure of $NS_3[2]$ is presented in Figure 3.

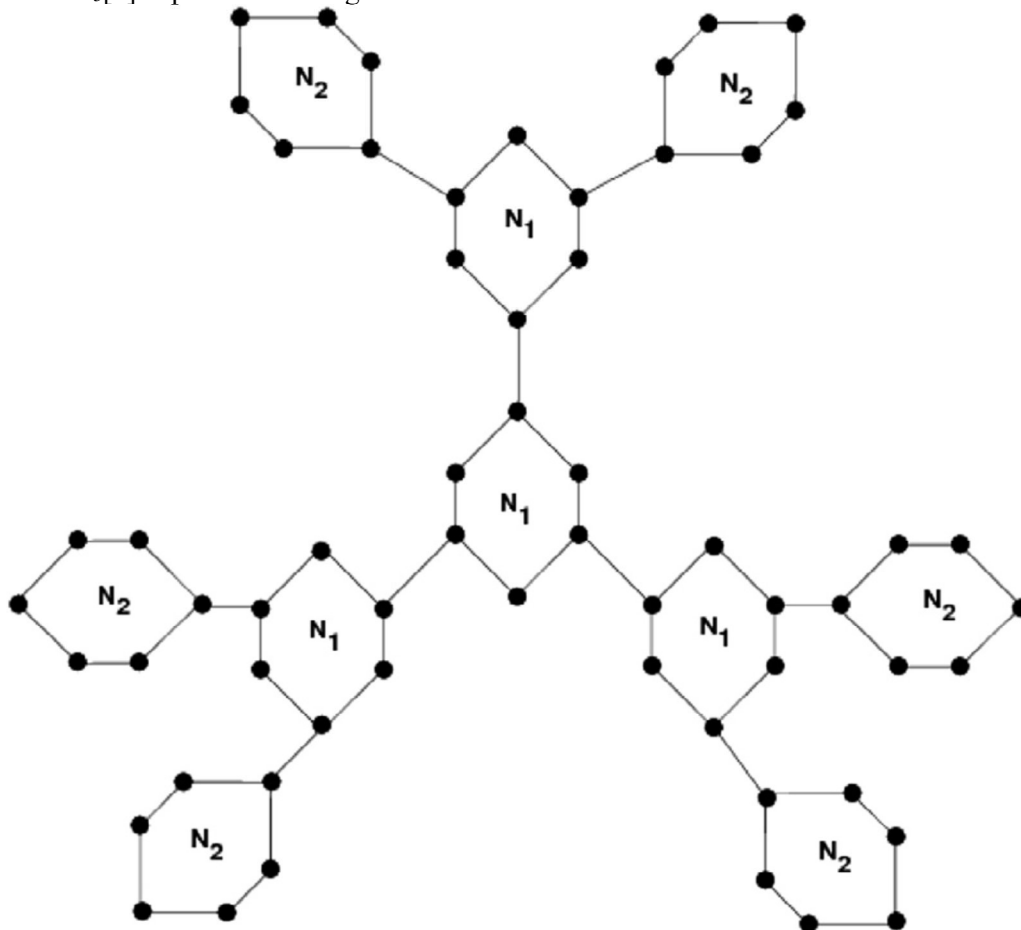


Figure 3. The structure of $NS_3[2]$

Let G be the molecular graph of $NS_3[n]$. By calculation, we obtain that G has $18 \times 2^n - 12$ vertices and $21 \times 2^n - 15$ edges. Also by calculation, we get that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degree of end vertices of each edge as given in Table 3.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4, 4)	(5, 4)	(5, 7)	(6, 7)	(7, 7)
Number of edges	3×2^n	3×2^n	3×2^n	$9 \times 2^n - 12$	$3 \times 2^n - 3$

Table 3. Edge partition of $NS_3[n]$ based on $S_G(u)$ and $S_G(v)$

We compute the multiplicative neighborhood general sum connectivity index of $NS_3[n]$.

Theorem 15. The multiplicative neighborhood general sum connectivity index of a dendrimer $NS_3[n]$ is given by

$$NM_1^a II(NS_3[n]) = 8^{a \times 3 \times 2^n} \times 9^{a \times 3 \times 2^n} \times 12^{a \times 3 \times 2^n} \times 13^{a(9 \times 2^n - 12)} \times 14^{a(3 \times 2^n - 3)}. \quad (5)$$

Proof: Let G be the molecular graph of $NS_3[n]$. By using definition and Table 3, we deduce

$$\begin{aligned} NM_1^a II(NS_3[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v)]^a \\ &= (4+4)^{a \times 2 \times 2^n} \times (5+4)^{a \times 3 \times 2^n} \times (5+7)^{a \times 3 \times 2^n} \times (6+7)^{a(9 \times 2^n - 12)} \times (7+7)^{a(3 \times 2^n - 3)} \\ &= 8^{a \times 3 \times 2^n} \times 9^{a \times 3 \times 2^n} \times 12^{a \times 3 \times 2^n} \times 13^{a(9 \times 2^n - 12)} \times 14^{a(3 \times 2^n - 3)}. \end{aligned}$$

We establish the following results from Theorem 15.

Corollary 15.1. The multiplicative first neighborhood index of $NS_3[n]$ is

$$NM_1 II(NS_3[n]) = 8^{3 \times 2^n} \times 9^{3 \times 2^n} \times 12^{3 \times 2^n} \times 13^{9 \times 2^n - 12} \times 14^{3 \times 2^n - 3}.$$

Corollary 15.2. The multiplicative first hyper neighborhood index of $NS_3[n]$ is

$$HNM_1 II(NS_3[n]) = 8^{6 \times 2^n} \times 9^{6 \times 2^n} \times 12^{6 \times 2^n} \times 13^{2(9 \times 2^n - 12)} \times 14^{2(3 \times 2^n - 3)}$$

Corollary 15.3. The modified multiplicative first neighborhood index of $NS_3[n]$ is

$${}^m NM_1 II(NS_3[n]) = \left(\frac{1}{8}\right)^{3 \times 2^n} \times \left(\frac{1}{9}\right)^{3 \times 2^n} \times \left(\frac{1}{12}\right)^{3 \times 2^n} \times \left(\frac{1}{13}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{14}\right)^{3 \times 2^n - 3}.$$

Corollary 15.4. The multiplicative neighborhood sum connectivity index of $NS_3[n]$ is

$$SNM II(NS_3[n]) = \left(\frac{1}{\sqrt{8}}\right)^{3 \times 2^n} \times \left(\frac{1}{3}\right)^{3 \times 2^n} \times \left(\frac{1}{\sqrt{12}}\right)^{3 \times 2^n} \times \left(\frac{1}{\sqrt{13}}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{\sqrt{14}}\right)^{3 \times 2^n - 3}.$$

Proof: Put $a = 1, 2, -1, -1/2$ in equation (5), we obtain the desired results.

We determine the multiplicative neighborhood general product connectivity index of $NS_3[n]$.

Theorem 16. The multiplicative neighborhood general product connectivity index of dendrimer $NS_3[n]$ is given by

$$NM_2^a II(NS_3[n]) = 16^{a \times 3 \times 2^n} \times 20^{a \times 3 \times 2^n} \times 35^{a \times 3 \times 2^n} \times 42^{a(9 \times 2^n - 12)} \times 49^{a(3 \times 2^n - 3)}. \quad (6)$$

Proof: Using the definition and Table 3, we obtain

$$\begin{aligned} NM_2^a II(NS_3[n]) &= \prod_{uv \in E(G)} [S_G(u) S_G(v)]^a \\ &= (4 \times 4)^{a \times 3 \times 2^n} \times (5 \times 4)^{a \times 3 \times 2^n} \times (5 \times 7)^{a \times 3 \times 2^n} \times (6 \times 7)^{a(9 \times 2^n - 12)} \times (7 \times 7)^{a(3 \times 2^n - 3)} \\ &= 16^{a \times 3 \times 2^n} \times 20^{a \times 3 \times 2^n} \times 35^{a \times 3 \times 2^n} \times 42^{a(9 \times 2^n - 12)} \times 49^{a(3 \times 2^n - 3)}. \end{aligned}$$

We establish the following results by using Theorem 9.

Corollary 16.1. The multiplicative second neighborhood index of $NS_3[n]$ is

$$NM_2 II(NS_3[n]) = 16^{3 \times 2^n} \times 20^{3 \times 2^n} \times 35^{3 \times 2^n} \times 42^{9 \times 2^n - 12} \times 49^{3 \times 2^n - 3}.$$

Corollary 16.2. The multiplicative second hyper neighborhood index of $NS_3[n]$ is

$$HNM_2 II(NS_3[n]) = 16^{6 \times 2^n} \times 20^{6 \times 2^n} \times 35^{6 \times 2^n} \times 42^{18 \times 2^n - 24} \times 49^{6 \times 2^n - 6}.$$

Corollary 16.3. The modified multiplicative second neighborhood index of $NS_3[n]$ is

$${}^m NM_2 II(NS_3[n]) = \left(\frac{1}{16}\right)^{3 \times 2^n} \times \left(\frac{1}{20}\right)^{3 \times 2^n} \times \left(\frac{1}{35}\right)^{3 \times 2^n} \times \left(\frac{1}{42}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{49}\right)^{3 \times 2^n - 3}.$$

Corollary 16.4. The multiplicative neighborhood product connectivity index of $NS_3[n]$ is

$$PNM II(NS_3[n]) = \left(\frac{1}{4}\right)^{3 \times 2^n} \times \left(\frac{1}{\sqrt{20}}\right)^{3 \times 2^n} \times \left(\frac{1}{\sqrt{35}}\right)^{3 \times 2^n} \times \left(\frac{1}{\sqrt{42}}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{7}\right)^{3 \times 2^n - 3}.$$

Corollary 16.5. The reciprocal multiplicative neighborhood product connectivity index of $NS_3[n]$ is

$$RPNMII(NS_3[n]) = 4^{3 \times 2^n} \times (\sqrt{20})^{3 \times 2^n} \times (\sqrt{35})^{3 \times 2^n} \times (\sqrt{42})^{9 \times 2^n - 12} \times 7^{3 \times 2^n - 3}$$

Proof: Put $a = 1, 2, -1, -1/2, 1/2$ in equation (6), we obtain the desired results.

Theorem 17. The general multiplicative neighborhood index of dendrimer $NS_3[n]$ is

$$NM^a II(NS_3[n]) = (2 \times 4^a)^{3 \times 2^n} \times (5^a + 4^a)^{3 \times 2^n} \times (5^a + 7^a)^{3 \times 2^n} \times (6^a + 7^a)^{9 \times 2^n - 12} \times (2 \times 7^a)^{3 \times 2^n - 3}$$

Proof: By using the definition and Table 3, we deduce

$$\begin{aligned} NM^a II(NS_3[n]) &= \prod_{uv \in E(G)} [S_G(u)^a + S_G(v)^a] \\ &= (2 \times 4^a)^{3 \times 2^n} \times (5^a + 4^a)^{3 \times 2^n} \times (5^a + 7^a)^{3 \times 2^n} \times (6^a + 7^a)^{9 \times 2^n - 12} \times (2 \times 7^a)^{3 \times 2^n - 3} \end{aligned}$$

From Theorem 17, we obtain the following result.

Corollary 17.1. The multiplicative F -neighborhood index of $NS_3[n]$ is

$$FNMII(NS_3[n]) = 32^{3 \times 2^n} \times 41^{3 \times 2^n} \times 74^{3 \times 2^n} \times 85^{9 \times 2^n - 12} \times 98^{3 \times 2^n - 3}$$

Theorem 18. The multiplicative inverse sum indeg neighborhood index of $NS_3[n]$ is

$$INMII(NS_3[n]) = 2^{3 \times 2^n} \times \left(\frac{20}{9}\right)^{3 \times 2^n} \times \left(\frac{35}{12}\right)^{3 \times 2^n} \times \left(\frac{42}{13}\right)^{9 \times 2^n - 12} \times \left(\frac{7}{2}\right)^{3 \times 2^n - 3}$$

Proof: By using the definition and Table 2, we deduce

$$\begin{aligned} INMII(NS_3[n]) &= \prod_{uv \in E(G)} \frac{S_G(u)S_G(v)}{S_G(u) + S_G(v)} \\ &= \left(\frac{4 \times 4}{4 + 4}\right)^{3 \times 2^n} \times \left(\frac{5 \times 4}{5 + 4}\right)^{3 \times 2^n} \times \left(\frac{5 \times 7}{5 + 7}\right)^{3 \times 2^n} \times \left(\frac{6 \times 7}{6 + 7}\right)^{9 \times 2^n - 12} \times \left(\frac{7 \times 7}{7 + 7}\right)^{3 \times 2^n - 3} \\ &= 2^{3 \times 2^n} \times \left(\frac{20}{9}\right)^{3 \times 2^n} \times \left(\frac{35}{12}\right)^{3 \times 2^n} \times \left(\frac{42}{13}\right)^{9 \times 2^n - 12} \times \left(\frac{7}{2}\right)^{3 \times 2^n - 3} \end{aligned}$$

Theorem 19. The multiplicative harmonic neighborhood index of $NS_3[n]$ is

$$HNMII(NS_3[n]) = \left(\frac{1}{4}\right)^{3 \times 2^n} \times \left(\frac{2}{9}\right)^{3 \times 2^n} \times \left(\frac{1}{6}\right)^{3 \times 2^n} \times \left(\frac{2}{13}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{7}\right)^{3 \times 2^n - 3}$$

Proof: By using the definition and Table 2, we derive

$$\begin{aligned} HNMII(NS_3[n]) &= \prod_{uv \in E(G)} \frac{2}{S_G(u) + S_G(v)} \\ &= \left(\frac{1}{4}\right)^{3 \times 2^n} \times \left(\frac{2}{9}\right)^{3 \times 2^n} \times \left(\frac{1}{6}\right)^{3 \times 2^n} \times \left(\frac{2}{13}\right)^{9 \times 2^n - 12} \times \left(\frac{1}{7}\right)^{3 \times 2^n - 3} \end{aligned}$$

Theorem 20. The multiplicative symmetric division neighborhood index of $NS_3[n]$ is

$$SDNMII(NS_3[n]) = 2^{6 \times 2^n - 3} \times \left(\frac{41}{20}\right)^{3 \times 2^n} \times \left(\frac{74}{35}\right)^{3 \times 2^n} \times \left(\frac{85}{42}\right)^{9 \times 2^n - 12}$$

Proof: By using the definition and Table 3, we deduce



$$\begin{aligned}
 SDNMII(NS_3[n]) &= \prod_{uv \in E(G)} \left(\frac{S_G(u)}{S_G(v)} + \frac{S_G(v)}{S_G(u)} \right) \\
 &= \left(\frac{4}{4} + \frac{4}{4} \right)^{3 \times 2^n} \times \left(\frac{5}{4} + \frac{4}{5} \right)^{3 \times 2^n} \times \left(\frac{5}{7} + \frac{7}{5} \right)^{3 \times 2^n} \times \left(\frac{6}{7} + \frac{7}{6} \right)^{9 \times 2^n - 12} \times \left(\frac{7}{7} + \frac{7}{7} \right)^{3 \times 2^n - 3} \\
 &= 2^{6 \times 2^n - 3} \times \left(\frac{41}{20} \right)^{3 \times 2^n} \times \left(\frac{74}{35} \right)^{3 \times 2^n} \times \left(\frac{85}{42} \right)^{9 \times 2^n - 12}.
 \end{aligned}$$

Theorem 21. The first and second multiplicative Gourava neighborhood indices of dendrimer $NS_3[n]$ is

$$(i) \quad NGO_1II(NS_3[n]) = 24^{3 \times 2^n} \times 29^{3 \times 2^n} \times 47^{3 \times 2^n} \times 55^{9 \times 2^n - 12} \times 63^{3 \times 2^n - 3}.$$

$$(ii) \quad NGO_2II(NS_3[n]) = 128^{3 \times 2^n} \times 180^{3 \times 2^n} \times 420^{3 \times 2^n} \times 546^{9 \times 2^n - 12} \times 686^{3 \times 2^n - 3}.$$

Proof: By using the definitions and Table 3, we derive

$$\begin{aligned}
 (i) \quad NGO_1II(NS_3[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v) + S_G(u)S_G(v)] \\
 &= (4 + 4 + 4 \times 4)^{3 \times 2^n} \times (5 + 4 + 5 \times 4)^{3 \times 2^n} \times (5 + 7 + 5 \times 7)^{3 \times 2^n} \\
 &\quad \times (6 + 7 + 6 \times 7)^{9 \times 2^n - 12} \times (7 + 7 + 7 \times 7)^{3 \times 2^n - 3} \\
 &= 24^{3 \times 2^n} \times 29^{3 \times 2^n} \times 47^{3 \times 2^n} \times 55^{9 \times 2^n - 12} \times 63^{3 \times 2^n - 3}.
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad NGO_2II(NS_3[n]) &= \prod_{uv \in E(G)} [S_G(u) + S_G(v)] S_G(u) S_G(v) \\
 &= [(4 + 4)4 \times 4]^{3 \times 2^n} \times [(5 + 4)5 \times 4]^{3 \times 2^n} \times [(5 + 7)5 \times 7]^{3 \times 2^n} \\
 &\quad \times [(6 + 7)6 \times 7]^{9 \times 2^n - 12} \times [(7 + 7)7 \times 7]^{3 \times 2^n - 3} \\
 &= 128^{3 \times 2^n} \times 180^{3 \times 2^n} \times 420^{3 \times 2^n} \times 546^{9 \times 2^n - 12} \times 686^{3 \times 2^n - 3}.
 \end{aligned}$$

CONCLUSION

In this study, we have introduced some multiplicative neighborhood indices. Furthermore, we have determined some new and old multiplicative neighborhood indices for nanocones and two types of dendrimers.

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